

WAVELET-BASED ALGORITHMS FOR MODELING MEDICAL SIGNALS AND FUNCTIONAL DATA

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Abstract. Objective. This article is devoted to the construction of wavelet models, which are considered important in function processing, and the use of Taylor series as a method of approximating analytical functions.

Objective. This article is devoted to the construction of wavelet models, which are considered important in function processing, and the use of Taylor series as a method of approximating analytical functions.

These models are built using Haar wavelets. It is important to reduce the number of coefficients required to approximate the total number of Binary segments (with a given accuracy) as a result of transformation of given signals using Haar wavelets in the form of a known function and analytical function. The process of calculating the coefficients using the Xarra wavelet is found without long operations. Only addition, scaling and transformation operations are used.

Methods. Haar wavelet, Taylor series, conversion wavelet, digital processing error, relative error.

Results. The obtained results show that it can be seen that the Haar wavelet gives higher accuracy compared to the Taylor series in the digital processing of the given signals in an analytical form.

Conclusion. In the problems of image recognition from waves, in the processing and synthesis of various signals such as speech, in the analysis of various images in nature (the color of the retina, radiography of the kidney, studying the surface properties of crystals and nano-objects, satellite images of clouds or planetary surfaces etc. can be used to study the properties of vortex fields and in other cases.

Key words: Haar wavelet, Taylor series absolute error, relative error, scaling function

Introduction. Currently, there are several types of wavelets. It is convenient to use uncomplicated methods to determine the coefficients of the function, one of the uncomplicated methods is the Haara wavelet, in which the coefficients of the function are determined without many operations only by adding, subtracting and scaling. It can also be noted that the preference of the type of wavelets also depends on the analysis of the input looking function or signal, because the scaling function is interpreted differently depending on the wavelet types. In medicine, wavelets are widely used when studying various signals, radiography of the kidney, properties of the surface of crystals and nano-objects, when studying the properties of gastroenterological signals and in other cases, interpolation. The waveforms of the Haar wavelet extend along the time axis along with the signal graph. A Haar wavelet plot often approximates a signal as a one-way waveform across the signal, which is good for compressing some signals. Its mathematical interpretation allows analysis of wave states at different frequencies. The amplitude of the graph of the Haar-wavelet function decreases to zero, forming oscillating waves.

Methods. Construction of a Haar wavelet. There are fast Haar wavelet transformation algorithms, and its orthogonal wavelets are widely used in solving practical problems.

An orthogonal Haar wavelet is expressed as follows:

$$har_k(x) = har_{pj}(x) = \begin{cases} +1 & x \in h_{pj}^- \\ -1 & x \in h_{pj}^+ \\ 0 & x \in h_{pj} \end{cases}$$

Considered as a wavelet in Haar bases. Haar wavelets attract the attention of experts for two reasons:

Reducing the number of coefficients required for approximation (with a given accuracy) compared to the total number of binary segments.

Absence of "long" operations in the process of calculating coefficients. Only add, scale, and transform operations are used.

In the digital processing of signals, wavelet functions are used to separate the details and local features of the signals, and scaling functions are used to approximate the signals. When choosing wavelet functions, special attention is paid to their characteristics, such as smoothness, carrier size, and the number of cases where their values are equal to zero.

The process of wavelet transformation of signals relies on the use of two types of functions: a wavelet function and a scaling function, which means that they are the same wavelet function shifted in time across the signal and time scale a is built by changing:

$$\psi_{ab}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right), \quad (a, b) \in \mathbb{R}, \quad \psi(t) \in L^2(\mathbb{R})$$

V^0 - that's all $[0,1]$ we define a set of invariant functions on the interval, that is, a set of linear vectors.

Then the following scaling function V_0 - belongs to the collection:

$$\phi(t) = \phi_{0,0}(t) = \begin{cases} 1, & 0 \leq t < 1 \\ 0, & \text{if} \end{cases} \quad (1)$$

(1) $i = 0$ zoom function when available.

$$\phi_{n,j}(t) = \begin{cases} 1, & \frac{j}{2^n} \leq t < \frac{j+1}{2^n}, \quad j = 0, 1, \dots, 2^n - 1 \\ 0, & \text{if} \end{cases} \quad (2)$$

$$V^0 \subset V^1 \subset \dots \subset V^n \subset \dots$$

(2) $i = n$ zoom function when available, here,

$$0 \leq 2^n t - j < 1, \quad \frac{j}{2^n} \leq t < \frac{j+1}{2^n}$$

is the interval of change of scaling functions, $\phi_{n,j}(t)$ and $\psi_{n,j}(t)$ are scaling functions related to V^n , in which there is a set of vectors with scalar multiplication, so these sets form the Euclidean space. In our case as a scalar multiplication

$$(f, g) = \int_0^1 f(t) g(t) dt \quad (3)$$

(3) we get the form using this formula C_n - coefficients of scaling functions are defined.

In that case

$$\phi_{n,j}(t) = \sqrt{2^n} \phi(2^n t - j), \quad j = 0, 1, \dots, 2^n - 1 \quad (4)$$

Using expressions (3) and (4), the coefficients of the Haar wavelet are found:

$$C_n = \int_0^1 \phi_n(x) f(x) dx \quad (5)$$

(5) Formula for finding coefficients of Haar wavelet.

$$f(x) \cong \sum_{n=0}^{\infty} C_n \phi_n(x)$$

Figure 1. Block diagram of function interpolation in Haara wavelet. Statement of the problem: Let's assume $t \times f \Rightarrow$ (function $t=0,25$ with step $[0,1]$ values are given, this function is required to be interpolated in the Haar wavelet. There is a way to solve the problem:

$$\begin{aligned} [u, w] &= \{t | u \leq t < w\} \\ \varphi_{[0,1]} t &= \begin{cases} 1, & \text{if } 0 \leq t < 1; \\ 0, & \text{else} \end{cases}; & \varphi_{[0,w]} t &= \begin{cases} 1, & \text{if } 0 \leq t < w; \\ 0, & \text{else} \end{cases}; \\ \varphi_{[u,w]} t &= \begin{cases} 1, & \text{if } u \leq t < w; \\ 0, & \text{else} \end{cases}; & \varphi_{[u,w]} t &= \begin{cases} s, & \text{if } u \leq t < w; \\ 0, & \text{else} \end{cases}; \end{aligned}$$

Generating the Haar wavelet function (6)

$$\begin{aligned} (6) \quad f &= s_j \varphi_{[t_j, t_{j+1}]} \\ &\approx s_0 \cdot \varphi_{[0,t]} + s_1 \cdot \varphi_{[t_1, t_2]} + \dots + s_{n-1} \cdot \varphi_{[t_{n-1}, t_n]} = \sum_{j=0}^{n-1} s_j \cdot \varphi_{[0,t]} \end{aligned}$$

Generating the Haar wavelet wavefunction (7)

$$\psi_{[u, w]} = \varphi_{[u, m]} - \varphi_{[m, w]} \quad (7)$$

Results. $t \times f \Rightarrow$ (let the function be given in the interval $[0,1]$ with a step of $t=0.25$, the number of compressions when interpolating this function in the Haara wavelet is equal to $n=4$.

$$\psi(t) = \sqrt{2} \begin{cases} 1, & 0 \leq t < \frac{1}{4}; \\ 0, & \text{else} \end{cases} \quad \psi(t) = 4 \begin{cases} 1, & \frac{1}{4} \leq t < \frac{1}{2}; \\ 0, & \text{else} \end{cases} \quad \psi(t) = 4 \begin{cases} 1, & \frac{1}{2} \leq t < \frac{3}{4}; \\ 0, & \text{else} \end{cases} \quad \psi(t) = 4 \begin{cases} 1, & \frac{3}{4} \leq t < 1; \\ 0, & \text{else} \end{cases}$$

To calculate the Haar wavelet coefficient, we calculate the following integral

$$\begin{aligned} C_0 &= \int_0^{1/4} f(t) \psi(t) dt = \int_0^{1/4} 4t dt = 2t^2 \Big|_0^{1/4} = 2 \left[\frac{1}{16} - 0 \right] = \frac{1}{8} \\ C_1 &= \int_{1/4}^{1/2} f(t) \psi(t) dt = \int_{1/4}^{1/2} 4t dt = 2t^2 \Big|_{1/4}^{1/2} = 2 \left[\frac{1}{4} - \frac{1}{16} \right] = \frac{3}{8} \\ C_2 &= \int_{1/2}^{3/4} f(t) \psi(t) dt = \int_{1/2}^{3/4} 4t dt = 2t^2 \Big|_{1/2}^{3/4} = 2 \left[\frac{9}{16} - \frac{1}{4} \right] = \frac{5}{8} \\ C_3 &= \int_{3/4}^1 f(t) \psi(t) dt = \int_{3/4}^1 4t dt = 2t^2 \Big|_{3/4}^1 = 2 \left[1 - \frac{9}{16} \right] = \frac{7}{8} \end{aligned}$$

$$f(t) \cong \sum_{i=0}^n C_i \psi(t); \quad f_0(t) \cong C_0 \varphi_0(t) = \frac{1}{8}; \quad f_1(t) \cong C_1 \varphi_1(t) = \frac{3}{8}; \quad f_2(t) \cong C_2 \varphi_2(t) = \frac{7}{8}; \quad f_3(t) \cong C_3 \varphi_3(t) = \frac{7}{8}$$

Based on the given model, the initial experimental data of the function was obtained and numerical processing was carried out on Haar wavelets (Fig.3).

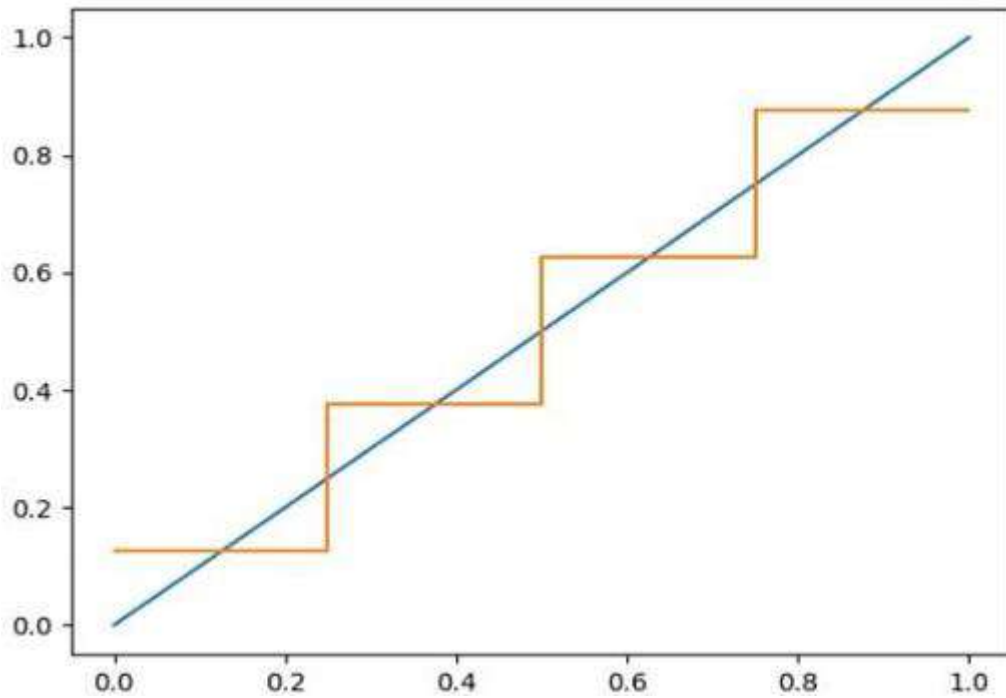


Figure 3. $f(x)=t$ Result of interpolation on Xaar wavelets. Haar's piece-invariant wavelet coefficients can also be determined by the following method. We can express the approximation and dn-difference coefficients of an-Haar through signal values in the following form

$$a_n = \frac{f_{2n-1} + f_{2n}}{2}, \quad n=1,2,3,\dots,N/2 \quad (8)$$

here $a_i=(a_1,a_2,\dots,a_{N/2})$ formula for determining average values. The differential value representation of the signal is

$$d_n = \frac{f_{2n-1} - f_{2n}}{2}, \quad n=1,2,3,\dots,N/2 \quad (9)$$

here $d_i=(d_1,d_2,\dots,d_{N/2})$ formula for determining difference values. These values generate two new signals: one of which is the original signal $a=\{a_n\}$, $n \in Z$ and the second is to restore the initial signal. $d=\{d_n\}$, $n \in Z$ Indeed

$$\begin{aligned} f_{2n-1} &= a_n + d_n; \\ f_{2n} &= a_n - d_n \end{aligned}$$

One of the main features of wavelets is the fast calculation algorithms for numerical processing of the calculated coefficients. Using the fast calculation algorithm of numerical processing of coefficients in Haara wavelet $f(t)=t^2$ we calculate the interpolation of the function in the interval $[0,1)$ with a step of 0.1. It is known that the number of compressions is equal to $n=10$.

$$t=(0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1),$$

$$F(t)=(0.0,0.01,0.04,0.09,0.16,0.25,0.36,0.49,0.64,0.81,1)$$

$$\begin{aligned}
a_0 &= \frac{f_{2n-1} + f_{2n}}{2} = \frac{0 + 0.01}{2} = 0.005; & a_1 &= \frac{f_{2n-1} + f_{2n}}{2} = \frac{0.01 + 0.04}{2} = 0.025; \\
a_2 &= \frac{f_{2n-1} + f_{2n}}{2} = \frac{0.04 + 0.09}{2} = 0.065; & a_3 &= \frac{f_{2n-1} + f_{2n}}{2} = \frac{0.09 + 0.16}{2} = 0.125; \\
a_4 &= \frac{f_{2n-1} + f_{2n}}{2} = \frac{0.16 + 0.25}{2} = 0.205; & a_5 &= \frac{f_{2n-1} + f_{2n}}{2} = \frac{0.25 + 0.36}{2} = 0.305; \\
a_6 &= \frac{f_{2n-1} + f_{2n}}{2} = \frac{0.36 + 0.49}{2} = 0.425; & a_7 &= \frac{f_{2n-1} + f_{2n}}{2} = \frac{0.49 + 0.64}{2} = 0.565; \\
a_8 &= \frac{f_{2n-1} + f_{2n}}{2} = \frac{0.64 + 0.81}{2} = 0.725; & a_9 &= \frac{f_{2n-1} + f_{2n}}{2} = \frac{0.81 + 1.00}{2} = 0.905;
\end{aligned}$$

On the basis of the given model, the initial experimental data of the function was obtained, and numerical processing was carried out on the Haar wavelet (Fig. 4).

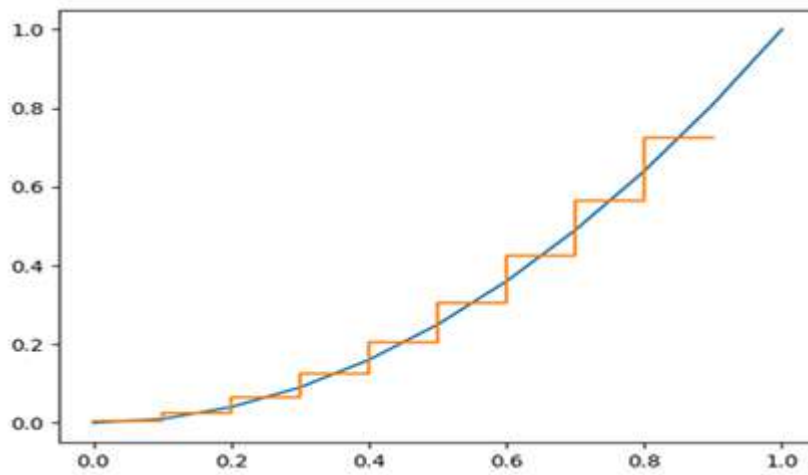


Figure 4. Using the algorithm of fast calculation of coefficients in the Haar wavelet $f(t)=t^2$ the result of interpolating the function in the interval $[0,1)$ with a step of 0.1.

Taylor series using Haara wavelet

The Haara wavelet is a very versatile mathematical tool that can be used to analyze, generate, and segment signals represented by functions or analytic functions. The use of Taylor series as a method of approximating analytic functions is one of the most common methods in applied mathematics. Using Taylor series with wavelets is another way to approximate analytical functions.

let's say $f(x)$ The function $x_0 \in R$ is one of the points

$$U_{\delta}(x_0) = \{x \in R: x_0 - \delta < x < x_0 + \delta; \quad \delta > 0\} \quad (10)$$

(10) Let have a derivative of any order around . This is the case $f(x)$ allows us to write the Taylor formula of the function:

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + r_n(x) \quad (11)$$

(11) In this $r_n(x)$ residual term. As long as $f(x)$ function $U_\delta(x_0)$ has a derivative

of any order, then

$$f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + \dots \quad (12)$$

(12) It is possible to look at the power series, (10) the coefficients of the power series are numbers, which $f(x)$ function and its derivatives x_0 expressed by the values at the point, (11) rank series $f(x)$ is called the Taylor series of the function.

In particular, $x_0=0$ (8) is the degree series

$$f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n + \dots = \sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{n!}x^n \quad (13)$$

(13) appears. Let's assume, $f(x)$ function is something $\in (-r, r)$ da $(r>0)$ has a derivative of any order, and its $x_0=0$ point Taylor series

$$f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n + \dots + \frac{f^{(n+1)}(0)}{(n+1)!}x^{n+1} + \dots \quad (14)$$

let it be This is the residual term of the series (14). $r_n(x)$ let's say:

$$f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n + r_n(x)$$

We find Taylor series of trigonometric functions.

let's say $f(x) = \sin x$ let it be Ravshanki $\forall x \in R, \forall n \in N$ at

$$|f(x)| \leq 1, |f^{(n)}(x)| \leq 1 \text{ being } f(0), f'(0) = 1, f^{(2n)}(0) = 0, f^{(2n+1)}(0) = (-1)^n \quad (n \in N)$$

will be.

So $f(x) = \sin x$ the function expands into a Taylor series and according to the formula (10).

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots \text{will be}$$

$$\sin(x) = 1 + 0.059964p(x) + 0.119712p(x+1) + 0.179030p(x+2)$$

Discussions. Aare the numerical performance errors of Haar $[a, b]$ defined in $f(x)$ be a continuous function. $[a, b]$ the segment

$$a \leq x_0 < x_1 < \dots < x_i < \dots < x_n \leq b$$

we can separate the nodes into points.

$$h = x_{i+1} - x_i = \text{const} \quad (15)$$

h - distance between nodes.

There are formulas for determining methodical errors of interpolation for polynomials of different degrees. For example, for polynomials of the zero degree (for piece-invariant wavelets), the error estimation formula is expressed as:

$$|P(x) - f(x)| \leq \frac{1}{2} \max |f'(x)|h$$

We present an estimate of the absolute and relative errors of digitizing the geophysical signal in Haar piece-invariant wavelets.:

$$\Delta_1 = \max_{a \leq x \leq b} |f(x_i) - \text{haar}(x_i)| = 0.02\%$$

Δ_1 - Absolute error of Haar's piece-invariant wavelets

Calculating the value of the function in the Taylor series and the Haar wavelet and estimating the absolute error (Table 1)

Table 1

Calculating the value of the function in the Taylor series and the Haar wavelet and estimating the absolute error

N₀	X[i]	$f(t) = \sin(6x)$ Taylor series	Haar	Absolute mistake
1	0.00000	0.000000	0.00082	0.04999
2	0.01000	0.059964	0.089838	
3	0.02000	0.119712	0.149371	
4	0.03000	0.179030	0.208366	
5	0.04000	0.237703	0.266611	
6	0.05000	0.295520	0.323897	
7	0.0600	0.352274	0.380017	
8	0.0700	0.407760	0.434770	
9	0.08000	0.461779	0.487958	
10	0.09000	0.514136	0.539389	
11	0.1000	0.564642	0.588880	
12	0.1100	0.613117	0.636251	
13	0.12000	0.659385	0.681332	
14	0.13000	0.703279	0.723961	
15	0.14000	0.744643	0.763985	
16	0.1500	0.783327	0.801259	
...	0.16000	0.819192	0.835650	
	...	...	...	
	0.97000	-0.446800	0.419575	
	0.98000	-0.392350	0.364419	
	0.99000	-0.336488	0.307952	

The first column of this table (Table 1) shows the initial values of the function, the second column shows the value of the Taylor series distribution, the third column shows the value of the coefficients of the square wavelet, and the fourth column shows the absolute error value.

Conclusion. In this research work, the digital performance model of Haar's wavelets of the given signal in the form of function and analytic function was evaluated. It can be seen from Table 1 that the absolute error for the number of signal nodes in the evaluation process is 0.0299944; was equal to the values. It was found that the error of digital processing in Haara wavelets is small, it can be concluded that the use of Haara wavelet in the process of digital processing of signals gives good results. This method can also be used to solve problems such as signal compression, medical signal filtering, and signal-to-noise separation..

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