

EYLER-LAGRANJNING BIR TURIDAGI KASR TARTIBLI DIFFERENSIAL TENGLAMASINI SONLI YECHISHNING BIR USULI

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Annotatsiya

Ushbu maqolada Riman-Liuvill va Kaputoning mos ravishda α , $0 < \alpha < 1$ tartibli chap va o'ng kasr tartibli hosilalarini o'z ichiga olgan Eyler-Lagranj kasr tartibli differensial tenglamalaridan birini Matlab dasturida dasturlash yordamida sonli yechishning bitta usulini taklif qilamiz.

Kalit so'zlar: Raqamli aniqlik darajasi, Kasr hosilalar, Matlab dasturida dasturlash

KIRISH

Kasr tartibli differensial tenglamalar keyingi yillarda olimlar uchun tobora qiziqarli bo'lib bormoqda. Ko'plab mualliflar allaqachon turli xil masalalarni o'rganib, turli chegaraviy shartlarga ega Eyler-Lagranj tenglamalarini shakllantirganlar [1-18]. Bizning ishimizda chegaraviy shartlar bilan α , $0 < \alpha < 1$ tartibli chap va o'ng kasr tartibli hosilalar kompozitsiyasini o'z ichiga olgan oddiy differensial tenglama ko'rib chiqilgan. Bu tipdagi tenglamalar Eyler-Lagranjning kasr tartibli differensial tenglamasi deb yuritiladi. Kasr tartibli differensial tenglamalar fizika, mexanika, elektrotexnika va h.k. kabi ba'zi sohalarida paydo bo'ladi... Bu turdagi tenglamalarni Laplas almashtirishi yoki eng kichik ta'sir prinsipidan foydalanish usuli kabi boshqa usullar bilan yechish mumkin [10,11]. Qo'zg'almas nuqtalar teoremasidan [6,7] foydalanuvchi analitik usul ham yechim beradi, ammo bu usul murakkabroq. Shuning uchun biz bu turdagi kasr differensiallarni yechishning yana bir sonli usulini taklif etamiz.

Tadqiqot metodologiyasi

Ushbu ish to'rt bo'limda taqdim etiladi: taqdim etadigan tenglamani yechish uchun zarur vositalar bo'lgan ta'riflarni eslatib o'tamiz. Sonli yechimni taklif qilamiz, avval tenglamada uchraydigan ikkala kasr operatorlarning sonli sxemalarini qidiramiz va so'ngra chiziqli tenglamalar sistemasini tuzish uchun tenglamaning kengaytirilgan shaklini aniqlaymiz, uni matritsa shakliga o'tkazamiz. Oxirgi paragrafda taqribiy sonli yechimlarni grafik usulda topish imkonini beruvchi Matlab tilida dasturlash va sonli yechimga misollar keltirilgan.

Dastlabki

Ushbu bo'limda ushbu sonli yechim uchun ba'zi foydali ta'riflarni keltiramiz. $[0, b]$ oraliqda tartibli Riman-Liuvill ma'nosidagi chap va o'ng kasrli integrallar mos ravishda quyidagicha aniqlanadi:

Ushbu bo'limda biz ushbu sonli yechim uchun foydali ba'zi ta'riflarni keltiramiz. $[0, b]$ oraliqda aniqlangan α -tartibli Riemann-Liouville ma'nosidagi chap va o'ng fraksional integrallar quyidagicha aniqlanadi:

(1) uchun: $t > 0$

$$I_{b-}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_t^b \frac{f(u)}{(u-t)^{1-\alpha}} du, \quad t < b$$

Bu yerda Γ — Eulerning gamma funksiyasi, va $b \in \mathbb{R}^+$.

Riemann-Liouville ta'rifiga ko'ra α -tartibli ($0 < \alpha < 1$) chap va o'ng fraksional hosilalar quyidagicha aniqlanadi:

(2) uchun: $t > 0$

$$D_{0+}^{\alpha} f(t) = \frac{d}{dt} I_{0+}^{1-\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{f(u)}{(t-u)^{\alpha}} du$$

$$(3) \text{ uchun: } t < b \quad D_{b-}^{\alpha} f(t) = -\frac{d}{dt} I_{b-}^{\alpha} f(t) = -\frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_t^b \frac{f(u)}{(u-t)^{\alpha}} du$$

Kaputo ma'nosidagi α -tartibli chap va o'ng fraksional hosilalar ham Riman–Liuvillning chap va o'ng fraksional integrallari orqali quyidagicha aniqlanadi.

$$(4) \text{ uchun: } t > 0 \quad {}^C D_{0+}^{\alpha} f(t) = I_{0+}^{1-\alpha} \left(\frac{d}{dt} f(t) \right) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f'(u)}{(t-u)^{\alpha}} du$$

$$(5) \text{ uchun: } t < b \quad {}^C D_{b-}^{\alpha} f(t) = -I_{b-}^{1-\alpha} \left(\frac{d}{dt} f(t) \right) = -\frac{1}{\Gamma(1-\alpha)} \int_t^b \frac{f'(u)}{(u-t)^{\alpha}} du$$

Riman–Liuvillning chap fraksional hosilasi bilan Kaputo'ning chap fraksional hosilasi orasidagi bog'lanish (ikkalasi ham α -tartibli) quyidagicha ifodalanadi:

$$(6) \text{ uchun: } t > 0 \quad D_{0+}^{\alpha} f(t) = {}^C D_{0+}^{\alpha} f(t) + \frac{t^{-\alpha}}{\Gamma(1-\alpha)} f(0)$$

$$(7) \text{ uchun: } t < b \quad D_{b-}^{\alpha} f(t) = {}^C D_{b-}^{\alpha} f(t) + \frac{(b-t)^{-\alpha}}{\Gamma(1-\alpha)} f(b)$$

Masalaning qo'yilishi

Ushbu ishda biz α -tartibli Eyler–Lagrange turidagi fraksional differensial tenglamaning bir turini o'rganamiz:

$$(8) \text{ tenglama} \quad D_{0+}^{\alpha} f(t) + \gamma f(t) = mg(t)$$

Bu yerda: f — noma'lum funksiya, g — berilgan funksiya, D_{0+}^{α} — Riemann–Liouville chap hosilasi, D_{1-}^{α} — Caputo o'ng hosilasi, $0 < \alpha < 1$, $t \in [0, 1]$.

$$f(0) = 0, f(1) = 0 \quad (9)$$

Sonli yechim

(8)-tenglamaning diskret ko'rinishini topish uchun $[0, 1]$ oraliq N ta teng qismga bo'linadi. Qadam uzunligi $h = \frac{1}{N}$ ga teng bo'ladi. Natijada ushbu oraliqda $(N+1)$ ta nuqta mavjud:

$$t_i, i = 0, 1, \dots, N$$

Bu yerda $t_0 = 0$, $t_N = 1$, va $t_i = ih = \frac{i}{N}$ nuqtalar o'sish tartibida joylashgan. Har bir t_i nuqtadagi funksiyaning diskret qiymati $f_i = f(t_i)$ deb belgilanadi. Shuningdek, funksiyaning hosilasi quyidagicha belgilanadi:

$$f'_i = \left(\frac{\partial f}{\partial t} \right)_{t=t_i}$$

(10) 1-tartibli chekli ayirmalar sxemasi quyidagicha yoziladi:

$$f'_i = \left(\frac{\partial f}{\partial t} \right)_{t=t_i} = \frac{f_{i+1} - f_i}{h}$$

Ushbu sxema har bir t_i nuqtada f funksiyaning hosilasining taxminiy qiymatini aniqlash imkonini beradi. Biz (8)-tenglamaning diskret ko'rinishini aniqlaymiz va hosil bo'lgan tenglamani matritsa ko'rinishiga o'tkazamiz. Quyida (8)-tenglamaning diskret ko'rinishini topish uchun bajariladigan asosiy bosqichlar keltirilgan.

Har bir turdagi fraksional hosilaning diskret ko'rinishini taqdim etamiz: avvalo Riman–Liuvillning chap fraksional hosilasi, so'ngra Kaputo'ning o'ng fraksional hosilasi, bu yerda tartib α , $0 < \alpha < 1$ [9, 14].

Riman–Liuvillning α -tartibli fraksional hosilasining t_i nuqtadagi yaqinlashgan (taxminiy) qiymati quyidagicha ifodalanadi:

Buning uchun Riman–Liuvillning chap fraksional hosilasi bilan Kaputo'ning chap fraksional hosilasi orasidagi (6)-munosabatdan foydalanamiz:

$$D_{0+}^{\alpha} f(t) = D_{0+}^{\alpha} f(t) + \frac{t_i^{-\alpha}}{\Gamma(1-\alpha)} f(0)$$

$$= \frac{1}{\Gamma(1-\alpha)} \int_0^{t_i} \frac{f'(u)}{(t_i-u)^{\alpha}} du + \frac{t_i^{-\alpha}}{\Gamma(1-\alpha)} f(0)$$

Hosilani chekli ayirmalar usuli (10) orqali yaqinlashtirish va integralni bo'laklarga ajratish natijasida quyidagi munosabatga ega bo'lamiz:

Ammo quyidagi integralni hisoblash: $\int_{t_j}^{t_{j+1}} \frac{1}{(t_i-u)^{\alpha}} du$ natijada quyidagini beradi:

$$\int_{t_j}^{t_{j+1}} \frac{1}{(t_i-u)^{\alpha}} du = \frac{(t_i-t_j)^{1-\alpha} - (t_i-t_{j+1})^{1-\alpha}}{1-\alpha} \quad (11)$$

Integralni (11)-munosabat yordamida almashtirib, quyidagini hosil qilamiz:

$$D_{0+}^{\alpha} f(t_i) \approx \frac{1}{\Gamma(1-\alpha)} \sum_{j=0}^{i-1} \frac{f_{j+1} - f_j}{h} \left[\frac{(t_i-t_j)^{1-\alpha} - (t_i-t_{j+1})^{1-\alpha}}{1-\alpha} \right] + \frac{t_i^{-\alpha}}{\Gamma(1-\alpha)} f_0$$

Soddalashtirib yozsak:

$$D_{0+}^{\alpha} f(t_i) \approx \frac{1}{(1-\alpha)\Gamma(1-\alpha)} \sum_{j=0}^{i-1} \frac{f_{j+1} - f_j}{h} [(t_i-t_j)^{1-\alpha} - (t_i-t_{j+1})^{1-\alpha}] + \frac{t_i^{-\alpha}}{\Gamma(1-\alpha)} f_0$$

$t_i = ih$ va $t_j = jh$ ekanligini hisobga olsak, quyidagini hosil qilamiz:

$$D_{0+}^{\alpha} f(t_i) \approx \frac{1}{\Gamma(2-\alpha)} \sum_{j=0}^{i-1} \frac{f_{j+1} - f_j}{h} [(ih-jh)^{1-\alpha} - (ih-(j+1)h)^{1-\alpha}] + \frac{(ih)^{-\alpha}}{\Gamma(1-\alpha)} f_0$$

$h^{1-\alpha}$ ni umumiy ko'paytuvchi sifatida ajratib olsak, quyidagi tenglikka ega bo'lamiz:

$$D_{0+}^{\alpha} f(t_i) \approx \frac{h^{1-\alpha}}{\Gamma(2-\alpha)} \sum_{j=0}^{i-1} \frac{f_{j+1} - f_j}{h} [(i-j)^{1-\alpha} - (i-(j+1))^{1-\alpha}] + \frac{(ih)^{-\alpha}}{\Gamma(1-\alpha)} f_0$$

Soddalashtirib yozsak:

$$D_{0+}^{\alpha} f(t_i) \approx \frac{h^{-\alpha}}{\Gamma(2-\alpha)} \sum_{j=0}^{i-1} (f_{j+1} - f_j) [(i-j)^{1-\alpha} - (i-(j+1))^{1-\alpha}] + \frac{(1-\alpha)i^{-\alpha}h^{-\alpha}}{\Gamma(2-\alpha)} f_0$$

yoki

$$D_{0+}^{\alpha} f(t_i) \approx \frac{h^{-\alpha}}{\Gamma(2-\alpha)} \left\{ \sum_{j=0}^{i-1} (f_{j+1} - f_j) [(i-j)^{1-\alpha} - (i-j-1)^{1-\alpha}] + (1-\alpha)i^{-\alpha}f_0 \right\}$$

$j = 0$ dan i gacha bo'lgan qiymatlar uchun f_j ning koeffitsiyentlari olinadi va ular

$h^{-\alpha} C_{i,j}$ ko'rinishida belgilanadi. Bu yerda:

$$C_{i,j} = \frac{1}{\Gamma(2-\alpha)} [(1-\alpha)i^{-\alpha} - i^{1-\alpha} + (i-1)^{1-\alpha}], j = 0$$

$$C_{i,j} = \frac{1}{\Gamma(2-\alpha)} [(i-j+1)^{1-\alpha} - 2(i-j)^{1-\alpha} + (i-j-1)^{1-\alpha}], 1 \leq j \leq i-1$$

$$C_{i,j} = \frac{1}{\Gamma(2-\alpha)}, j = i$$

Endi φ funksiyasi uchun α -tartibli Kaputo o'ng fraksional hosilasining yaqinlashgan ifodasi quyidagicha beriladi:

$${}^c D_{1-}^{\alpha} \varphi(t) = -I_{1-}^{1-\alpha} D\varphi(t) = -\frac{1}{\Gamma(1-\alpha)} \int_t^1 \frac{\varphi'(u)}{(u-t)^{\alpha}} du$$

nuqtada, chekli ayirmalar usuli (10) formulasi va integralni (11) bo'laklarga ajratishdan foydalanib, quyidagini hosil qilamiz:

$$D_{1-}^{\alpha} \varphi(t_i) \approx -\frac{1}{\Gamma(1-\alpha)} \sum_{j=i}^{N-1} \frac{\varphi_{j+1} - \varphi_j}{h} \int_{t_j}^{t_{j+1}} \frac{1}{(u-t_i)^{\alpha}} du$$

Integralni hisoblab, quyidagiga ega bo'lamiz:

$$D_{1-}^{\alpha} \varphi(t_i) \approx -\frac{1}{\Gamma(1-\alpha)} \sum_{j=i}^{N-1} \frac{\varphi_{j+1} - \varphi_j}{h} \left[\frac{(t_{j+1} - t_i)^{1-\alpha} - (t_j - t_i)^{1-\alpha}}{1-\alpha} \right]$$

Endi $t_i = ih$ va $t_j = jh$ ni qo'yib, yaqinlashgan ifoda quyidagicha yoziladi:

$$D_{1-}^{\alpha} \varphi(t_i) \approx -\frac{1}{\Gamma(1-\alpha)} \sum_{j=i}^{N-1} \frac{\varphi_{j+1} - \varphi_j}{h} \left[\frac{((j+1)h - ih)^{1-\alpha} - (jh - ih)^{1-\alpha}}{1-\alpha} \right]$$

$h^{1-\alpha}$ ni umumiy ko'paytuvchi sifatida ajratsak:

$$D_{1-}^{\alpha} \varphi(t_i) \approx -\frac{h^{1-\alpha}}{\Gamma(1-\alpha)} \sum_{j=i}^{N-1} \frac{\varphi_{j+1} - \varphi_j}{h} \left[\frac{(j+1-i)^{1-\alpha} - (j-i)^{1-\alpha}}{1-\alpha} \right]$$

Soddalashtirib yozsak:

$$D_{1-}^{\alpha} \varphi(t_i) \approx -\frac{h^{-\alpha}}{(1-\alpha)\Gamma(1-\alpha)} \sum_{j=i}^{N-1} (\varphi_{j+1} - \varphi_j) [(j-i+1)^{1-\alpha} - (j-i)^{1-\alpha}]$$

yoki

$$D_{1-}^{\alpha} \varphi(t_i) \approx \frac{h^{-\alpha}}{\Gamma(2-\alpha)} \sum_{j=i}^{N-1} (\varphi_{j+1} - \varphi_j) [(j-i)^{1-\alpha} - (j-i+1)^{1-\alpha}]$$

(12)

$i \leq j \leq N-1$ bo'lganda, φ_j funksiyaning koeffitsiyentlari $h^{-\alpha} d_{i,j}$ orqali belgilanadi. Bu yerda:

$$d_{i,j} = \frac{1}{\Gamma(2-\alpha)}, j = i$$

$$d_{i,j} = \frac{1}{\Gamma(2-\alpha)} [(j-i+1)^{1-\alpha} - 2(j-i)^{1-\alpha} + (j-i-1)^{1-\alpha}], i+1 \leq j \leq N-1$$

$$d_{i,j} = \frac{1}{\Gamma(2-\alpha)} [(N-i-1)^{1-\alpha} - (N-i)^{1-\alpha}], j = N$$

Shunday qilib, ikki operatorni kompozitsiya qilish orqali boshlang'ich (8)-tenglamani quyidagi diskret ko'rinishda yozish mumkin:

$$\begin{cases} f_0 = 0, \\ \frac{h^{-2\alpha}}{\Gamma(2-\alpha)} \sum_{j=i}^N d_{i,j} \sum_{k=0}^j c_{j,k} f_k + r f_i = m g_i, (13) \quad 1 \leq i \leq N-1, \\ f_N = 0 \end{cases}$$

$i = 1$ dan $N - 1$ gacha bo'lgan qiymatlar uchun, har bir iga mos holda quyidagi yoyilgan ko'rinishga ega bo'lamiz:

$$a_{i,0}f_0 + a_{i,1}f_1 + \dots + a_{i,i}f_i + \dots + a_{i,N}f_N = m g_i (14)$$

Bu yerda $a_{i,k}$ koeffitsiyentlar ($0 \leq k \leq N$) quyidagicha aniqlanadi:

$$\begin{aligned} a_{i,k} &= \left(\frac{1}{N}\right)^{-2\alpha} \sum_{j=i}^N d_{i,j} c_{j,k}, \quad 0 \leq k < i \\ a_{i,k} &= \left(\frac{1}{N}\right)^{-2\alpha} \sum_{j=i}^N d_{i,j} c_{j,k} + r, \quad k = i (15) \\ a_{i,k} &= \left(\frac{1}{N}\right)^{-2\alpha} \sum_{j=k}^N d_{i,j} c_{j,k}, \quad i+1 \leq k \leq N \end{aligned}$$

Shunday qilib, noma'lum f_i lar uchun $N + 1$ ta tenglamadan iborat quyidagi tizimni yozish mumkin:

$$\begin{cases} f_0 = 0, \\ a_{1,0}f_0 + a_{1,1}f_1 + \dots + a_{1,N}f_N = m g_1, \\ \vdots \\ \vdots \\ a_{N-1,0}f_0 + a_{N-1,1}f_1 + \dots + a_{N-1,N}f_N = m g_{N-1}, \\ f_N = 0 \end{cases} \quad (16)$$

Ushbu tizimni matritsa ko'rinishida quyidagicha yozish mumkin:

$$A_{i,k} \times F_i = G_i (17)$$

Bu yerda:

$A_{i,k}$ tizimdagi f_i lar oldidagi koeffitsiyentlardan tashkil topgan matritsa,

F_i noma'lumlar vektori,

G_i o'ng tomondagi qiymatlar (erkin hadlar) vektori.

Koeffitsiyentlar matritsasi quyidagi ko'rinishga ega:

$$A = \begin{pmatrix} 1 & 0 & \dots & 0 \\ a_{1,0} & a_{1,1} & \dots & a_{1,N} \\ \vdots & \vdots & \dots & \vdots \\ a_{N-1,0} & a_{N-1,1} & \dots & a_{N-1,N} \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

Bu matritsa tizimdagi f_i lar koeffitsiyentlaridan tashkil topgan. bu yerda F_i — noma'lum f_i qiymatlaridan tashkil topgan ustun vektor bo'lib, $0 \leq i \leq N$.

$$F_i = \begin{pmatrix} f_0 \\ f_1 \\ \vdots \\ f_{N-1} \\ f_N \end{pmatrix}$$

$$G_i = \begin{pmatrix} 0 \\ mg_1 \\ \dots \\ mg_{N-1} \\ 0 \end{pmatrix}$$

bu yerda G_i — tenglamaning o‘ng tomonini (erkin hadlar vektori) ifodalaydi. Mazkur koeffitsiyentlarni quyida keltiriladigan

MATLAB dasturlash tili yordamida hisoblash mumkin.

MATLAB dasturlash muhiti asosida hisoblash va sonli misollar

Ushbu dasturlashda, avvalo No' zgaras bo‘lib, α o‘zgaradigan holat ko‘rib chiqiladi, so‘ngra α o‘zgaras bo‘lib, No' zgaradigan holat tahlil qilinadi. Maqsad N va α qiymatlarining natijalarga ta‘sirini grafik ko‘rinishda aniqlashdir. Quyida keltirilgan dastur yordamida $c_{j,k}$, $d_{i,j}$ va $a_{i,k}$ koeffitsiyentlari bosqichma-bosqich hisoblanadi hamda $f(N, \alpha)$ funksiyasini grafik tarzda tasvirlash imkoniyati yaratiladi. Ushbu bo‘limda hisoblashlarning sonli natijalari keltiriladi. Tenglamalarning yechimlarini olishda quyidagi parametrlar qabul qilinadi: No' zgaras bo‘lganda: $N = 100$ va $N = 200$, $\alpha \in \{0.1, 0.2, 0.8, 0.9, 1\}$ α o‘zgaras bo‘lganda: $\alpha = 0.1$, $\alpha = 0.8$, $\alpha = 0.9$, $N \in \{200, 300, 500, 1000\}$.

Bu yerda g funksiyasi, r va m konstantalar dastur ichida beriladi. Ushbu qiymatlarni dasturda o‘zgartirish mumkin. Mazkur dasturiy misolda quyidagi funksiyalar va konstantalar qabul qilingan:

$$g(t) = 2\pi^2 \sin(\pi t), r = \pi^2, m = 2\pi^2 \quad (17)$$

funksiyasini hisoblash dasturi

```
function f = varalpha(N,alpha)
% N = input('value of N=')
% alpha = input('value of alpha=')
c = zeros(N, N+1);
% c koeffitsiyentlarini hisoblash
% *****
for j = 1:N
    for k = 0:j
        if k == 0
            ta = 1/gamma(2-alpha);
            tb = (1 - alpha)*j^(-alpha);
            tc = (j)^(1-alpha);
            td = (j - 1)^(1-alpha);
            c(j,k+1) = ta * (tb - tc + td);
        elseif k < j
            ta = 1/gamma(2-alpha);
            tb = (j - k + 1)^(1-alpha);
```

```

        tc = (j - k)^(1-alpha);
        td = (j - k - 1)^(1-alpha);
        c(j,k+1) = ta * (tb - 2*tc + td);
    else % k == j
        c(j,k+1) = 1/gamma(2-alpha);
    end
end
end
% Calculation of d
% *****
d = zeros(N,N);
for i=1:N-1
    for j=i:N
        if j == i && j ~== N
            d(i,j) = 1/gamma(2-alpha);
        elseif j < N
            tad = 1/gamma(2-alpha);
            tbd = (j - i + 1)^(1-alpha);
            tcd = (j - i)^(1-alpha);
            tdd = (j - i - 1)^(1-alpha);
            d(i,j) = tad * (tbd - 2*tcd + tdd);
        else
            t1d = (N - i - 1)^(1 - alpha);
            t2d = (N - i)^(1 - alpha);
            d(i,j) = (t1d - t2d)/gamma(2-alpha);
        end
    end
end
% matrix coefficients
%*****
b = 1;
a = zeros(N+1,N+1);
ce = (b / N)^(-2*alpha);
r = pi^2;
for i = 1 : N - 1
    for k = 0 : N
        S = 0;
        if k < i
            for j = i : N
                S = S + d(i,j)*c(j,k+1);
            End
        a(i+1,k+1) = ce * S;
        elseif k == i
            for j = i : N
                S = S + d(i,j)*c(j,k+1);

```

```

        end
        a(i+1,k+1) = ce * S + r;
    else
        for j = k : N
            S = S + d(i,j)*c(j,k+1);
        end
        a(i+1,k+1) = ce * S;
    end
end
end
cc = 1;
bb = zeros(N+1,1);
for i = 2 : N
    bb(i,1) = 2*pi^2*sin(pi*(i/N));
end
a(1,1)=1;
a(N+1,N+1)=1;
bb(1,1)=0;
bb(N+1,1)=0;
f = a\bb;
for i=1:N+1
    t(i)=i.*(1/N)
end
plot(t,f)
grid minor

```

Natijalar tahlili

Oldingi grafik natijalarni tahlil qilganimizda quyidagilar kuzatiladi:

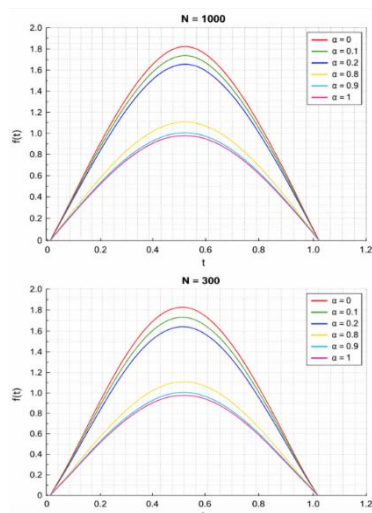
Agar α o'zgarmas bo'lib, N o'zgaradigan bo'lsa, barcha egri chiziqlar shaklan bir xil bo'ladi va ular deyarli ustma-ust tushadi.

Biroq, yaqinlashgan yechimning maksimal qiymati $t_i = 0.5$ nuqtada bir xil bo'lib qoladi, va Nortgani sari qiymatlar kamayadi. Agar N o'zgarmas bo'lib, α o'zgaradigan bo'lsa: egri chiziqlarning shakli o'zgarmaydi, lekin ular α ning turli qiymatlari uchun farqlanadi. f_i funksiyaning maksimal qiymatlari t_i ga bog'liq holda, turli N qiymatlarida deyarli bir xil bo'lib qoladi. Shuningdek, quyidagilar kuzatiladi:

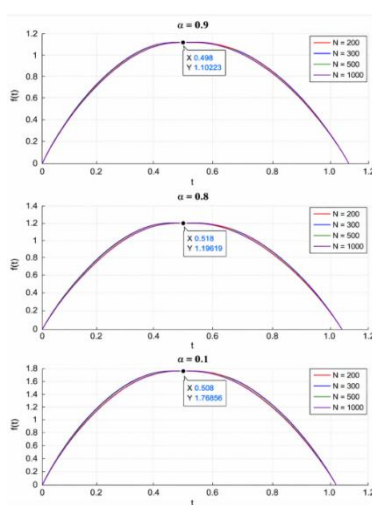
$\alpha \rightarrow 0$ bo'lganda, grafiklar $\alpha = 0$ ga mos egri chiziqqa yaqinlashadi

$\alpha \rightarrow 1$ bo'lganda, grafiklar $\alpha = 1$ ga mos egri chiziqqa yaqinlashadi

(1 va 2-rasmlar bilan tasdiqlanadi)



1-rasm. N o'zgarmas bo'lib, α o'zgaradigan holat uchun taxminiy yechimlar.



2.Rasm. α o'zgarmas bo'lib, N o'zgaradigan holat uchun taxminiy yechimlar.

Xulosa

Ushbu ishimizda biz MATLAB dasturlash muhiti yordamida sonli usuldan foydalandik. Bu usul matritsa koeffitsiyentlarini aniqlash orqali funksiyaning berilgan nuqtadagi taxminiy yechimlarini topish imkonini beradi. Hisoblashlar ikki xil holatda amalga oshirildi: ayrim parametrlar o'zgarmas bo'lib, boshqalari o'zgaruvchi bo'lgan holatlar ko'rib chiqildi. Bizning maqsadimiz ushbu usulni turli sohalaridagi masalalarga, jumladan fan va mexanika kabi yo'nalishlarda qo'llashdan iborat.

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